Geometry of Non-expanding Horizons and Their Neighborhoods

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Abstract

This is a contribution to MG9 session BHT4. Certain geometrically distinguished frame on a non-expanding horizon and in its space-time neighborhood, as well as the Bondi-like coordinates are constructed. The construction provides free degrees of freedom, invariants, and the existence conditions for a Killing vector field. The reported results come from the joint works with Ashtekar and Beetle [2].

In the quasi-local theory of black holes proposed recently by Ashtekar [1] a BH in equilibrium is described by a 3-dimensional null cylinder $\mathcal{H}$ generated in space-time by null geodesic curves intersecting orthogonally a space-like, 2-dimensional closed surface S. The standard stationarity of space-time requirement is replaced by the assumption that the cylinder has zero expansion, that is $\mathcal{H}$ is a *non-expanding horizon*. This implies, upon the week and the dominant energy conditions, that the induced on $\mathcal{H}$ (degenerate) metric tensor q is Lie dragged by a null, geodesic flow tangent to $\mathcal{H}$. The *geometry* induced on $\mathcal{H}$ consists of the metric tensor q and the induced covariant derivative $\mathcal{D}$. It is enough for the mechanics of $\mathcal{H}$ [1]. The geometry of a non-expanding horizon is characterized by local degrees of freedom. They are an arbitrary 2-geometry of the null generators space S, the rotation scalar, and certain tangential 'radiation' evolving along the horizon.

In the standard, Kerr-Newman case, the event horizon is equipped with a null Killing vector field. In our general non-expanding horizon case, however, a Killing vector field may not exits at all. Our first goal is a geometric condition which distinguishes a null vector field ℓ_0 tangent to $\mathcal{H}$ and which is satisfied by the Killing vector field whenever it exists. We a made extra assumptions about the stress energy tensor at $\mathcal{H}$ that are satisfied for the Maxwell and/or scalar and/or dylaton fields. The condition distinguishing the null vector field ℓ_0 was obtained by making as many components of the tensor $[\ell, \mathcal{D}]^a_{bc}$ defined on $\mathcal{H}$ as possible zero, as we vary ℓ. But here we give a more geometric definition of this choice. Due to the evolution equations of $\mathcal{D}$ along $\mathcal{H}$, there is a unique extension $(\tilde{\mathcal{H}}, \tilde{q}, \tilde{\mathcal{D}})$ of $(\mathcal{H}, q, \mathcal{D})$ in an affine parameter along the null geodesics. We claim, that generically $\tilde{\mathcal{H}}$ admits a unique global crossection S_0 such that its expansion in the transversal null direction orthogonal to S_0 (this information is contained in $\tilde{D}$) is zero everywhere on S_0. Given the crossection S_0, there is a unique null vector field ℓ_0 vanishing identically on S_0 and such that $\mathcal{D}_{\ell_0}\ell_0 = \kappa_0\ell_0$, $\kappa_0 \neq 0$ being a constant. Fixing some value $\kappa_0(q, \mathcal{D})$ determines ℓ_0 completely. The shear of S_0 vanishes in the null transversal direction orthogonal to $\tilde{\mathcal{H}}$, iff ℓ_0 generates a symmetry of the geometry $(q, \mathcal{D})$. The commutator $[\mathcal{L}_{l_0}, \mathcal{D}]$ represents the tangential radiation, and $\mathcal{H}$ is not a Killing horizon unless the comutator is zero.

The *rotation 1-form potential* ω_0 of ℓ_0 is defined by $\mathcal{D}\ell_0 \;=\; \omega_0 \otimes \ell_0$. We define a *good cut* as a space-like section of $\mathcal{H}$ such that the pullback of ω_0 thereon is a harmonic 1-form. The good cuts define a foliation of $\mathcal{H}$ invariant with respect to the flow of ℓ_0, owing to $\mathcal{L}_{\ell_0}\omega_0 \;=\; 2d\kappa_0 \;=\; 0$.

Given ℓ_0 and the good cuts foliation, we determine a null frame $(m_0, \bar{m}_0, n_0, \ell_0)$ by using another null vector n_0 orthogonmal to the lives requiring $n_{0\mu}{\ell_0}^\mu \;=\; -1$, and $\mathrm{Re}{m_0}^\mu K_{,\mu} = 0$ where K is the Gauss curvature of $\mathcal{H}$, generically non-constant.

In a neighborhood of $\mathcal{H}$, the good cuts foliation and the distinguished ℓ_0 define a unique geodesic extension of the vector field n_0. It is used to extend the foliation and frame to the neighborhood.

The applications and results of this construction are a) invariants of the horizon and of the neighborhood, b) invariant characterization and true degrees of freedom of a horizon and of its neighborhood in the vacuum or Maxwell and/or scalar and/or dylaton case, c) classification of the symmetric isolated horizons, d) necessary and sufficient conditions for the existence

of a Killing vector field, and the control on the space-times not admitting a Killing vector field.

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References

[1] Ashtekar A., Beetle C., Dreyer O., Fairhurst S., Krishnan B., Lewandowski J., Wiśniewski J., *Phys.Rev Let.***85**3564(2000), gr-qc/0006006, See the contribution by Olaf Dreyer in the same session

[2] Ashtekar A., Beetle C., Lewandowski J. *Space-Time Geometry of Isolated Horizons I,II*, in preparation